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Procedural Versions of Set Definitions

Let X and Y be subsets of a universal set U and suppose x and y are elements of U.

1. $x \in X \cup Y \quad \Leftrightarrow \quad x \in X$ or $x \in Y$
2. $x \in X \cap Y \quad \Leftrightarrow \quad x \in X$ and $x \in Y$
3. $x \in X - Y \quad \Leftrightarrow \quad x \in X$ and $x \notin Y$
4. $x \in X^c \quad \Leftrightarrow \quad x \notin X$
5. $(x, y) \in X \times Y \quad \Leftrightarrow \quad x \in X$ and $y \in Y$

Example 6.2.1 Proof of a Subset Relation

Prove Theorem 6.2.1(1)(a): For all sets A and B, $A \cap B \subseteq A$.

Solution We start by giving a proof of the statement and then explain how you can obtain such a proof yourself.

Proof:

Suppose A and B are any sets and suppose x is any element of $A \cap B$. Then $x \in A$ and $x \in B$ by definition of intersection. In particular, $x \in A$. Thus $A \cap B \subseteq A$.

The underlying structure of this proof is not difficult, but it is more complicated than the brevity of the proof suggests. The first important thing to realize is that the statement to be proved is universal (it says that for *all* sets A and B, $A \cap B \subseteq A$). The proof, therefore, has the following outline:

Starting Point: Suppose A and B are any (particular but arbitrarily chosen) sets.

To Show: $A \cap B \subseteq A$

Now to prove that $A \cap B \subseteq A$, you must show that

$$\forall x, \text{ if } x \in A \cap B \text{ then } x \in A.$$

But this statement also is universal. So to prove it, you

suppose x is an element in $A \cap B$

and then you

show that x is in A.

Filling in the gap between the "suppose" and the "show" is easy if you use the procedural version of the definition of intersection: To say that x is in $A \cap B$ means that

x is in A and x is in B.

This allows you to complete the proof by deducing that, in particular,

x is in A,